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An Implementation of the π -Calculus on the .NET

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Abstract

Here, we show how to refine the grammar of the π -calculus so that we can implement the π -calculus by decomposing a given process expression into a hierarchy whose nodes are responsible to communicate links and execute the basic actions specified in the process expression. The nodes in the hierarchy add new child nodes when they need to handle lower-level process expressions. They are removed from the hierarchy when the processes assigned to them are completed. Hence, the hierarchy for executing a process expression of the π -calculus grows and shrinks dynamically at runtime. We implement the nodes with concurrently executing threads in the managed C++ on Microsoft .NET. The communication requests from the nodes are coordinated and resolved with a class named CommunCenter. Thus, we present an operational semantics for the π -calculus and implement the semantics in the .NET.

1 INTRODUCTION

The π -calculus, introduced by Milner et al. [Milner et al. 1989, Milner et al. 1992], models the changing connectivity of a communication system, in which links to sites are sent between sites and used by the sites for communication. It has been studied extensively as a theoretical model and shows a powerful capacity in modeling data structures, mobile communication systems, and other real-world systems [Milner 1999, Sangiorgi and Walker 2001].

Here, we present an approach to implement the π -calculus. In the approach, we realize a π -calculus process expression by decomposing the expression into a hierarchy, whose nodes manage, coordinate, and perform the basic actions that are specified in the process expression. When the process expression is executed, new nodes are added to the hierarchy to account for lower-level process expressions. Existing nodes in the hierarchy whose tasks are completed are removed from the hierarchy. Thus, the execution of the π -calculus expression is realized and visualized with a dynamically changing hierarchy. To implement the approach, we associate a node of the hierarchy with an execution thread in the managed C++ language on the Microsoft .NET Framework and realize the hierarchy with the communicating concurrent threads. A class named CommunCenter is used to match input and output requests from the threads. The approach represents a feasible framework for a full-fledged distributed system that is based on the π -calculus to specify the dynamically changing connectivity of the components in the system.

The π -calculus is commonly regarded as an algebraic language. It uses algebraic expressions of processes to specify the communication tasks to be realized by the components in a communication system and use operators to manipulate the algebraic expressions. Our approach to implementing the π -calculus is based on a reformulation of the syntactic rules of the π -calculus so that we can unambiguously decompose a process expression into a hierarchical structure, which allows a natural divide-conquer style in implementing the π -calculus with the .NET execution threads. This approach is also different from the author's another paper to be published in Journal of Object Technology [Li 2005]. Here, we reformulate the syntax of the π -calculus and represent a given process expression with a dynamically changing hiearchy of nodes. The nodes can be realized with concurrently communicating threads and the execution of a process expression by a node may dynamically dispatch execution threads. Thus, a communication system that executes a π -calculus process expression can be realized with a distributed system, whose components are dynamically changed, added, and removed. The work presented in [Li 2005] transforms a π -calculus process expression into a Java program, which is to be interpreted by the Java virtual machine.

This paper is organized as follows. We introduce the π -calculus and reformulate the syntactic and semantic rules of the π -calculus in Section 2. Based on the reformulated syntax of the π calculus, we present an ASCII language for coding process expressions of the π -calculus and decomposing the coded expressions into hierarchies in Section 3. Based on the hierarchical representation of a given process expression, we describe an operational semantics for handling the hierarchyies in order to execute the π -calculus process expression in Section 3. We describe the concurrency mechanism supported by the .NET in Section 4 and specify how to apply the mechanism to implement the π -calculus in Section 5. The paper is concluded in Section 6.

In the following presentation, blue font is used to present source code, brown font presents syntactic constructs appeared in grammar rules, and *green italic font* shows non-terminal symbols in grammar rules and in running text. Particularly, a pair of braces followed by an asterisk {}* is used in grammar rules to enclose a syntactic component that may be repeated zero or multiple times. The source code files of the .NET implementation of the presented approach to realizing the π -calculus process expressions are compressed into a zip file, which is attached with this submission. The executable code of the implementation for Windows is also attached with the submission. Its usage will be described in Section 5.

2 THE π -CALCULUS

As a computational apparatus, the π -calculus has syntactic and semantic rules. Here, we briefly introduce the grammar and the operational semantics of the π -calculus based on the monograph [Sangiorgi and Walker 2001]. Then, we reformulate the syntax of the π -calculus for the purpose of an implementation of the π -calculus. Detailed elaboration of the π -calculus as an algebraic notation can be found in the monographs [Milner 1999, Sangiorgi and Walker 2001].

An Introduction to the π -Calculus

In the π -calculus [Milner 1999, p. 87], we can assume an infinite set N of channel names, which are denoted by lowercase letters x, y, z, ... with possible subscripts. Process expressions in the

 π -calculus are composed of basic actions, summations, and processes. A (*basic*) action π takes one of the following forms [Sangiorgi and Walker 2001, p. 11]:

π	::=	$x(\vec{y})$	receive names from channel x through parameters $\vec{y} = y_1,, y_k$		
		$\overline{x}\langle \vec{z} \rangle$	send $m \ge 0$ channel names $\vec{z} = z_1,, z_m$ into channel x		
		τ	execute an unobservable internal (silent) action		
		$[x = y] \pi$	enable action π if names x and y are same		

A summation M takes one of the forms:

M ::= 0	do nothing (an inaction process)
π . P	execute action π and proceed to process P
$M_{1} + M_{2}$	choose one of the summations M_1 and M_2 to execute

A process P takes one of the forms

P ::= M	execute summation M		
$P_1 \mid P_2$	execute processes P_1 and P_2 concurrently		
$v \ \overline{z} \ P$	declare $n \ge 1$ channel names $\vec{z} = z_1,, z_n$ for process P		
!P	supply (an infinite number of) copies of process P		

In the above syntactic rules, the π -calculus uses operators '.', '+', '|', and '!' to compose process expressions. With the dot operator '.' as a process constructor, a process expression π . Pschedules a sequential execution of a basic action π and a process P so that the action π must be completed successfully before the process P can be started. We say the process P is guarded by the action π [Milner 1999, p. 87]. The addition operator '+' in process expression $M_1 + M_2$ is used to separate the alternative processes M_1 and M_2 so that an execution of one of the two processes renders the other void. Due to the precedence of composition operator '!' over addition operator '+', neither of the process addends M_1 and M_2 in the choice expression $M_1 + M_2$ can show a top-level occurrence of the operator '!'. The composition operator '!' in process expression $P_1 | P_2$ is used to dictate a parallel execution of the processes P_1 and P_2 . The repeat operator '!' in process expression !P indicates that copies of the process P can be supplied on demand. Expression !P is structurally equivalent to the composite process expression P | !P [Sangiorgi and Walker 2001, p. 20].

The π -calculus uses symbol 0 to denote a nullifying process or a null, which does nothing but terminating the execution of the process that encounters the null 0. In the π -calculus, the null 0 is used as a placeholder symbol to indicate positions in *process contexts*, which are used as templates to generate process expressions [Sangiorgi and Walker 2001, p. 19]. The above syntactic rules allow an occurrence of the null 0 to be followed by any basic actions as well as by an operator '+' or '|'. The execution of a process expression can terminate naturally when it exhausts the actions specified in the expression, and action expressions that follow null 0 will be ignored in execution. We could safely ignore occurrences of the nullifying process 0 from process expressions [Sangiorgi and Walker 2001, p. 20] and simplify a process expression such as $\nu w \overline{x} \langle w \rangle$.

In the π -calculus [Milner 1999, p. 88; Sangiorgi and Walker 2001, p. 16], a pair of parentheses can be used to enclose a process expression to clarify the scope of an operator such as '+', '|', or '.'. For a valid process expression R, which must be terminated with null 0, we regard the expression (R) as a basic action. An occurrence of an operator such as '+', '|', or '.' inside (R) is an embedded but not top-level occurrence of the operator with respect to the process expression P that includes the basic action expression (R). A channel name x declared in the subexpression (R) in process expression P hides the same channel name x that is declared in P prior to the basic action (R).

In the π -calculus, a *prefix* of a process expression is a ν -prefix $\nu \vec{z}$ in process expression $\nu \vec{z} P$, an action-prefix π in $\pi . P$, or a match-prefix [x = y] in $[x = y] \pi . P$. By convention [Sangiorgi and Walker 2001, p. 16], a prefix has a higher precedence than operators '+' and '|'. Hence, the scope of the restriction ν -prefix $\nu \vec{z}$ in process $\nu \vec{z} P$ is not extended over any top-level occurrence of operator '+' or '|' in the process expression P. The choice operator '+' has a higher precedence than the composition operator '|'. For example, in the process expression $\nu z \vec{x} \langle z \rangle . (\nu z \vec{x} \langle z \rangle . 0) . z(\nu) . 0 + \vec{z}(\nu) . 0$, the first ν -prefix νz restricts the second and the fifth occurrence of the name z, the second ν -prefix νz restricts the fourth occurrence of the name z, and the last occurrence of the name z is a free name in the whole process expression.

If we regard the above set of syntactic rules as a grammar Θ with P as its goal, the grammar is neither LR(1) nor LL(1) as defined in [Aho et al. 1985] but ambiguous. For example, the π -calculus process expression $v z \overline{x} \langle z \rangle .0 | x(y) .0$ can be derived from the goal P through two different top-down leftmost derivations:

$$P \rightarrow P_1 | P_2 \rightarrow v z P_3 | P_2 \rightarrow v z \pi P_5 | P_2 \rightarrow v z \overline{x} \langle z \rangle P_5 | P_2 \rightarrow v z \overline{x} \langle z \rangle M_1 | P_2$$

$$\rightarrow v z \overline{x} \langle z \rangle 0 | P_2 \rightarrow v z \overline{x} \langle z \rangle 0 | M_2 \rightarrow v z \overline{x} \langle z \rangle 0 | \pi P_6$$

$$\rightarrow v z \overline{x} \langle z \rangle 0 | x(y) P_6 \rightarrow v z \overline{x} \langle z \rangle 0 | x(y) M_3 \rightarrow v z \overline{x} \langle z \rangle 0 | x(y) 0$$

$$P \rightarrow v z P_4 \rightarrow v z P_3 | P_2 \rightarrow \dots$$
 (Omitted derivation is same as the above derivation.)

A Reformulation of the π -Calculus

Following the above presentation of the syntactic and semantic specification of the π -calculus, we can characterize π -calculus process expressions by introducing several notions. We say that an occurrence of an operator '+' or '|' in a process expression is a *top-level* occurrence if it is not enclosed within any pair of parentheses; otherwise, it is an *embedded* occurrence. A *choice* process expression is a process expression that does not contain any top-level occurrence of the composite operator '|'. A *sequential* process expression is a process expression that does not contain any top-level occurrence of the operator '|' or '+'. We extend the syntactic rules of the π -calculus recursively with a basic action expression (R) for any process expression R. We regard the expression (R) as an instance of the symbol π in the above syntactic rules. Note that the basic action expression in the π -calculus since it is not ended with an inaction symbol 0.

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We shall use the following recursive syntactic specification of π -calculus process expressions to construct and parse the process expressions. Here, the term *basic action expression*, or simply action expression, represented with symbol A denotes inaction 0, an expression as denoted with the symbol π in the above syntactic rules, or an expression (R) for a process expression R.

S1. A *composite* process expression P consists of one or more choice process expressions M that are separated with the composite operator '|'; i.e., the syntax of a composite process expression is defined with rule

 $P ::= M \{ |M \}^*$

S2. A *choice* process expression M consists of one or more sequential process expressions S that are separated with the choice operator '+'; i.e.,

 $M ::= S \{ +S \}^*$

S3. A *sequential* process expression S consists of basic actions that are separated with the dot operator '.' and each of which is possibly prefixed with a sequence of matches, restrictions, and/or repeat symbols. It is ended with a null action 0. In the following rules, we use symbol E to denote a match, and D a restriction.

 $S ::= \{ B . \}^* 0$ $B ::= \{ E | D | ! \}^* A$ $A ::= 0 | \pi | (P)$

We shall use the term a *binding expression* γ to refer to either an expression $x(\vec{y})$, which consists of an input action $x(\vec{y})$ followed by the prefixing operator '.', or a v-prefix (restriction) $v \vec{y}$. The binding expression γ , which is either $x(\vec{y})$. or $v \vec{y}$, in a sequential process expression γS binds (the scope of) the channel names \vec{y} to the sequential process S and we call γ a binding prefix of the sequential process γS . For simplicity of presentation [Sangiorgi and Walker 2001, p. 47, Convention 1.4.10], we require that if the sequential process expression γS includes a binding expression γ' inside S, the channel names bound by γ' be different from those bound by γ . In a sequential process expression, names that are not bound by any binding expression are *free*. We require the bound names in a sequential process expression be different from the free names in the expression. For example, the sequential process expression $v z \overline{x} \langle z \rangle . (v z \overline{x} \langle z \rangle . 0) . z (v) . 0$ does not satisfy the first requirement. In the π -calculus [Sangiorgi and Walker 2001, p. 15], two process expressions P and Q are α -convertible if we can transform one of them to the other by changing bound names, and α -convertible process expressions are regarded as equal. For example, we can transform the sequential process expression $v z \overline{x} \langle z \rangle . (v z \overline{x} \langle z \rangle . 0) . z(v) . 0$ to an equal sequential process expression $v \ z \ \overline{x} \langle z \rangle$.($v \ y \ \overline{x} \langle y \rangle$.0).z(v).0 by applying α -conversion.

The π -calculus realizes a communication between a pair of input and output processes with a substitution of the output arguments received from the output process for the input parameters used in the input process. A *substitution* is a function $\sigma: \mathcal{N} \to \mathcal{N}$ that maps names to names and that is the identity except on a finite set of names. If a substitution σ maps names \vec{a} to names \vec{b} ,

the mapping can be denoted with $\sigma \vec{a} = \vec{b}$. Two sequential processes $\overline{x} \langle \vec{y} \rangle . S_1$ and $x(\vec{z}) . S_2$ can *communicate* if and only if the number of output arguments \vec{y} equals the number of input parameters \vec{z} and the mapping that maps each parameter z_i in the list \vec{z} to the corresponding argument y_i in the list \vec{y} can be extended to a substitution σ , which can be the identity on names that are not in list \vec{z} . The communication will transform the processes $\overline{x} \langle \vec{y} \rangle . S_1$ and $x(\vec{z}) . S_2$ to processes S_1 and $S_2\sigma$, respectively. The expression $S_2\sigma$ denotes a process expression resulting from S_2 by replacing occurrences of parameters z_i with corresponding arguments σz_i . Thus, the channel names \vec{y} are passed from a process $\overline{x} \langle \vec{y} \rangle . S_1$ to another process S_2 .

Assume a choice process expression $\tau . S + M'$. After the sequential process expression $\tau . S$ in the choice process expression is chosen to execute, the front internal action τ is executed and the choice process is reduced to the sequential process S. Thus, executing the silent action τ in the first subprocess $\tau . S$ in the given choice process expression $\tau . S + M'$ renders the alternative process M' void.

3 REALIZING π -CALCULUS PROCESSES

Process Expressions

To code π -calculus processes in ASCII, we use keywords in, out, and tao to signify the operational semantics of input, output, and internal actions. We use keyword new to introduce channel names declared by a ν -prefix restriction. The language for coding π -calculus processes in ASCII is specified with the following grammar, which follows the reformulated syntax of the π -calculus and which has the goal *composite_process* for producing process expression.

<composite_process></composite_process>	::=	<choice_process> <composite_process> </composite_process></choice_process>
		<choice_process></choice_process>
<choice_process></choice_process>	::=	<sequential_process> + <choice_process></choice_process></sequential_process>
		<sequential_process></sequential_process>
<sequential_process></sequential_process>	::=	<action> . <sequential_process></sequential_process></action>
		new < <i>identifier</i> > < <i>names</i> > < <i>sequential_process</i> >
		[<identifier> = <identifier>] <sequential_process> </sequential_process></identifier></identifier>
		! <sequential_process></sequential_process>
		0
<action></action>	::=	in <identifier> <names></names></identifier>
		out <identifier> <names> </names></identifier>
		tao
		(<composite_process>)</composite_process>
		0
<names></names>	::=	<identifier> <names></names></identifier>
		ε

Note that due to the occurrences of the null 0 in both the production rules for non-terminal symbols *sequential_process* and *action*, the above grammar is still ambiguous. However, the grammar allows us to distinguish *composite_process*, *choice_process*, and *sequential_process* expressions uniquely in a given process expression. In the above grammar, non-terminal *composite_process* corresponds to a π -calculus process *P*, which may show top-level occurrences of operators '|' and '+'. The top-level occurrences of the composite_process expression are used to decompose the *composite_process* expression into choice process, which are represented with the non-terminal *choice_process* and which may show top-level occurrences of the choice operator '+' in a *choice_process* expression decompose the *choice_process* expression into sequential process, which are represented with the non-terminal *sequential_process*. A *sequential_process* expression does not include any top-level occurrence of operator '|' or '+'. It must be ended with an inaction 0.

In the above grammar, the symbol ε denotes an empty sequence of identifiers. Keyword new in a *sequential_process* expression represents a ν -prefix by declaring channel names, which are denoted with *<identifier> <names>*. We call an expression that is composed of the keyword new and one or more channel names a *new-term*, an expression in form [*<identifier> = <identifier>*] a *match-term*, and an expression !, which consists of the single character '!', a *repeat-term*. We use the notion of a *term* to denote a new-, match-, or repeat-term.

The keyword in in an expression in x y1 ... yk with $k \ge 0$ introduces an input action $x(\vec{y})$ with bound variables $\vec{y} = y1 \dots yk$. The keyword out in an expression out x z1 ... zk introduces an output action $\overline{x}\langle \vec{z} \rangle$. Keyword tao denotes an internal action τ . A composite action (*R*) in the π -calculus is represented with (<*composite_process>*) in the above grammar. We regard the inaction 0 as a special type of action and use the term *action* to refer to an input, output, silent, composite action, or an inaction. A *sequential_process* expression consists of a sequence of actions that are separated with the dot operator '.' and is ended with an inaction 0. Each of the actions in a *sequential_process* expression may be prefixed with a sequence of terms. For example, we can code the input and output actions $x(y x_2 x_1)$ and $\overline{x}\langle x_1 x_2 y \rangle$ with action expressions in x y x2 x1 and out x x1 x2 y, respectively. The π -calculus process expression $\nu y \vec{x}\langle y\rangle.0$ can be coded in *sequential_process* expression **new** y **out** x y.0.

Process Hierarchy

Given a process expression P coded in the above ASCII language, we construct a hierarchy, denoted as $\Im(P)$ or simply \Im , to represent the expression P as follows:

- H0. The given expression P is regarded as a *composite_process* expression and denoted with the root node of \mathfrak{I} .
- H1. For each node ρ in \Im that denotes a *composite_process* expression Q which is not 0, we denote each of the *choice_process* expressions M in the expression Q with a child node ρ' of ρ in \Im .
- H2. For each node ρ' in \Im that denotes a *choice_process* expression M, we denote each of the *sequential_process* expressions S in the expression M with a child node ρ'' of ρ' in \Im .
- H3. For each node ρ " in \Im that represents a *sequential_process* expression S, we can handle the

terms in front of the first action in the sequential process S at runtime. (The runtime handling of the terms will be detailed after we specify the data structures of the nodes in hierarchy \Im .) Then, if the first action in S is a composite action (R), we add a unique child node for the node ρ " to denote the *composite_process* expression R and apply the rule H1 recursively with Q equal to R; otherwise, the node ρ " is a leaf in \Im .

We can characterize the hierarchy \Im as follows:

- T1. A composite_process node ρ has at least one choice_process child ρ' . A non-root composite_process node ρ that represents expression R has a unique sequential_process parent whose first action is (R).
- T2. A *choice_process* node ρ' has a unique *composite_process* parent and has at least one *sequential_process* child ρ'' .
- T3. A sequential_process node ρ " has a unique choice_process parent and represents a sequential_process expression S. The node ρ " is a leaf in \Im if the first action in S is not a composite action.

Each node in the hierarchy \Im keeps track of its parent and children. In addition to the parentchild relation, a *sequential_process* node ρ " also holds the following data structures, which are modified at runtime.

- A list procExpr of characters for holding the *sequential_process* expression S to be executed by the node ρ",
- A list declaredNames of channel names declared in new-terms that have been handled by the node ρ",
- A collection repeatedNodes of *sequential_process* nodes, which are created by node ρ" when the node handles repeat-terms, and
- A mapping inputValue that maps input parameters z to node-name pairs (ζ", y) such that an input action performed by ρ" has received the output argument y from another *sequential_process* node ζ" through the input parameter z. The performance of input and output operations are specified in the following subsections.

The character list or string procExpr in ρ " is initialized to a given *sequential_process* expression when the node ρ " is created. The data structures declaredNames, repeatedNodes, and inputValue are initially empty when ρ " is created.

When a *sequential_process* node ρ " evaluates a match term $[n_1 = n_2]$, it needs to decide whether names n_1 and n_2 are free names, input parameters received or channel names declared by node ρ " or by ancestors of ρ ". We use symbol inputValuePlus at the *sequential_process* node ρ " to denote a mapping that combines mapping inputValue at node ρ " with the mappings inputValue at its *sequential_process* ancestors; i.e., value inputValuesPlus(n) at node ρ " equals the value inputValue(n) at ρ " if n is in the domain of the mapping inputValue at node ρ " or equals inputValue(n) at lowest ancestor ζ " of ρ " if n is in the domain of the mapping inputValue at ζ ". If the name n is free with respect to the whole hierarchy, we shall construct a value (0, n), which consists of the pointer 0 and name n, for the function inputValuesPlus(n).

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An Operational Semantics

When a process expression P is executed, we parse the expression P automatically through the steps H0-H4 to construct the hierarchy $\Im(P)$, which consists of *composite_process* nodes ρ , *choice_process* nodes ρ' , and *sequential_process* nodes ρ'' . The nodes collaborate to execute tasks required by the given process expression P. We specify the tasks for different types of node as follows.

- S1. A *composite_process* node ρ monitors its *choice_process* child nodes ρ' so that it is removed from the hierarchy \Im when all its children complete their tasks.
- S2. A *choice_process* node ρ' manages its *sequential_process* children. If any one of its children progresses by performing a basic action, the node ρ' removes its other *sequential_process* children and, thus, realizes a choice. The *choice_process* node is removed from the hierarchy \Im when it has no more children.
- S3. A sequential_process node ρ " maintains a sequential_process expression procExpr. It handles the terms that are in front of the first action in procExpr and, then, processes the first action δ in the expression procExpr as follows:
- If δ is a composite action (R), the hierarchy ℑ is extended as described in Step H3 by adding a *composite_process* child node for the node ρ". The new child holds the *composite_process* expression R. The node ρ" can progress only after its child completes the execution of the process R and is removed from the hierarchy ℑ.
- If δ is an input or output action, the node ρ" is a leaf in hierarchy ℑ and it requests the class CommunCenter to perform the input or output action. Node ρ" continues its operation only after the class CommunCenter fulfils the input or output request. As described in Step S2, the continuation of the node ρ" will request the *choice_process* ancestors of ρ" to remove their *sequential_process* children except the *sequential_process* ancestors of node ρ".
- If δ is a silent action tao, the node ρ" progresses and requests its *choice_process* ancestors to remove their *sequential_process* children except the *sequential_process* ancestors of the node ρ".
- If δ is inaction 0, the node ρ" is removed from the hierarchy ℑ. As described in Steps S1 and S2, the removal of node ρ" may trigger removal of its ancestors from ℑ.

We now describe how a *sequential_process* node ρ " handles the new-, match-, and repeatterms $\varphi_1, \ldots, \varphi_k$ with $k \ge 0$ that are in front of the first action δ in the *sequential_process* expression procExpr. The node ρ " handles the terms $\varphi_1, \ldots, \varphi_k$ by modifying the data structures declaredNames, inputValue, and repeatedNodes as follows. After handling a front term in the expression procExpr, node ρ " removes the term from the expression procExpr. We shall use the notation ρ "(n) for a name n to represent the value inputValuesPlus(n).

- A new-term new y1 ... yk is handled by appending the declared channel names y1, ..., yk into the list declaredNames in node ρ ".
- A repeat-term ! is handled by creating a *sequential_process* node ζ" such that the values in the data structures procExpr, declaredNames, and inputValue in the new node ζ" are copied from the corresponding data structures in node ρ" but the collection repeated-Nodes in the new node ζ" is empty. The new node ζ" is inserted into the list repeated-

Nodes in node ρ ". It will be added into the hierarchy after an action in node ρ " is performed. Thus, we realize a π -calculus transformation from expression !P to P|!P.

• A match-term $[n_1 = n_2]$ is handled by comparing values $\rho''(n_1)$ and $\rho''(n_2)$ of the names n_1 and n_2 . If the values $\rho''(n_1)$ and $\rho''(n_2)$ are equal, the node ρ'' removes the match-term from the expression procExpr and continues executing the expression procExpr; otherwise, node ρ'' terminates its execution and is removed from the hierarchy \Im .

The collection repeatedNodes holds *sequential_process* nodes that have no parent or child. Data structure inputValue is a dictionary that maps input parameters to output arguments received through input actions by the node ρ ". When the node ρ " uses input action in x y1 ... yk to communicate with an output action **out** x z1 ... zk of another node ζ ", the key-value pairs (y1, ζ "(z1)), ..., (yk, ζ "(zk)) will be added into the dictionary inputValue in node ρ ". Thus, parameters y1, ..., yk and arguments ζ "(z1), ..., ζ "(zk) values are related by the function inputValue. After the communication, nodes ρ " and ζ " discard their performed actions from their procExpr expressions and continue the processing of their remaining actions.

Process Communication

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We use class CommunCenter to coordinate communication requests proposed by the concurrently executing *sequential process* nodes in the hierarchy \Im . The class CommunCenter defines lists inputRequests and outputRequests to hold the input and output requests that cannot be fulfilled immediately. Particularly, when a sequential_process node ρ " submits an input request r_0 to the class CommunCenter, the class CommunCenter first searches the list outputRequests for an output request r_i such that r_i uses the same communication link as r_0 and the length of the parameter list of r_0 is equal to the length of the argument list of r_1 . If the search succeeds and the input and output requests are proposed by nodes n_0 and n_1 , respectively, the class CommunCenter allows the nodes n_0 and n_1 to realize their actions as follows; if the search fails, the class CommunCenter saves the input request r_0 into the inputRequests list. The node n_0 realizes its input action by extending the function inputValue so that the function inputValue maps the parameters in r_0 to the corresponding arguments in r_1 . As described in Step S3, the node n_0 also asks its *choice_process* ancestors to remove their sequential_process childrens except the ancestors of the node n_0 . Node n_1 also asks its choice_process ancestors to do the same thing. Similarly, when a sequential_process node p" submits an output request, the class CommunCenter searches the list inputRequests to decide if the output request can be communicated immediately. If the output request can be fulfilled for a saved input request, the output and input actions are performed by the sequential_process nodes that proposed the output and input requests, respectively; otherwise, the output request is placed into the list outputRequests.

4 MULTITHREADING IN THE .NET

We now introduce the multithreading mechanism of the .NET Framework for implementing the nodes in a heirarchy $\Im(P)$ for a π -calculus process expression P. We also describe the standard .NET classes Monitor and Mutex, which are designed for thread synchronization. We shall use the facilities provided in the class Monitor to synchronize the execution threads that execute *composite_process* and *sequential_process* expressions. Accesses to the class CommunCenter are also synchronized with the class Monitor. The class Monitor can be replaced with the class Mutex in the implementation of the π -calculus.

.NET Threads

In addition to its main thread, an application in the .NET can create one or more concurrent execution threads, which are represented with objects of the standard class Thread. To create a thread, we need a ThreadStart delegate to encapsulate a method that will be executed by the created thread. For example, the following statement creates a thread cpThread. The thread encpsulates a *composite_process* node referenced by variable cp and a named processing defined in class CompositeProcess.

The method processing defined in class CompositeProcess performs the activities of a *composite_process* node as described in the operational semantics in Section 3. The above statement encapsulates an object of class CompositeProcess pointed at by the first argument cp of the constructor of the ThreadStart delegage. To encapsulate a static method in a ThreadStart object, the first argument of the constructor for the created ThreadStart delegate is the pointer 0. We can use the standard method GetHashCode in expression cpThread->GetHashCode() to return a value for identifying the thread cpThread. In our implementation of the π -calculus, we also create threads to encapsulate objects of class SequentialProcess.

In the execution of an application, a thread is always in one or a combination of the states represented with values of the standard enumeration type ThreadState. The initial state of a newly created thread such as cpThread is Unstarted. After the instance method Start() of thread cpThread is executed in statement

```
cpThread->Start();
```

thread cpThread enters the Running state, in which the encapsulated method processing is executed concurrently with other threads.

When a running thread executes method Sleep(), Wait(), or Join(), the thread enters state WaitSleepJoin and its execution is blocked. We can use an integer argument s to invoke the static method Sleep of class Thread so that the current thread will be blocked for the number s of milliseconds before it regains the Running state. The method Sleep() invoked in the following statement with integer 0 as argument just yields the rest of the thread's CPU time slice:

```
Thread::Sleep(0);
```

Methods Wait() and Join() specify a thread to wait for or join. For example, a running thread can execute the statement

```
cpThread->Join();
```

to enter state WaitSleepJoin. The thread will be in the WaitSleepJoin state until the target thread cpThread terminates.

A thread in state WaitSleepJoin leaves the state and reenters state Running when the sleep period expires, the waited or joined thread calls method Pulse or PulseAll, or another thread executes the function Interrupt for the thread.

A thread can invoke the method Suspend() or Abort() of another thread to request the other thread to enter a Suspended or Stopped state. A Suspended thread can reenter the Running state when another thread invokes its Interrupt method. A thread in the Stopped state can no longer execute and is subject to garbage collection.

Class Monitor

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We can use the class Monitor, defined in namespace System: :Threading, to synchronize multiple threads that access an object. A thread uses the static methods Enter, TryEnter, and Exit of class Monitor to get or release the lock of an object, which is pointed at by a method argument. For example, we shall use an object cc of the class CommunCenter to incapsulate the lists inputRequests and outputRequests, which are dicussed in Section 3. A thread that implements a *sequential_process* node can get the exclusive lock of the object cc by executing the statement

```
Monitor::Enter(cc);
```

If the lock of the intended object cc is not possessed by another thread, the thread will hold the lock and continue its execution; otherwise, its execution is blocked until the lock is released by the other thread that is holding the mentioned lock. A thread releases the lock of object cc through statement

```
Monitor::Exit(cc);
```

The method TryEnter invoked in the bool expression

```
Monitor::TryEnter(cc)
```

does not block the current thread, which is evaluating the expression. Its evaluation returns either true or false immediately depending on whether or not the current thread can get the lock of object cc.

Note that an execution of the static method Enter by a thread may block the thread but not change the Running state of the thread to WaitSleepJoin. If a Running thread that holds the lock of object cc evaluates the static method Wait in statement

```
Monitor::Wait(cc);
```

it releases the lock and enters the state WaitSleepJoin for the lock. A thread in state WaitSleepJoin for the lock of object cc cannot get the lock unless another thread that is holding the lock invokes the static method Pulse or PulseAll to signal the thread and then releases the lock by evaluating the Exit method of class Monitor.

For example, the implementation of the π -calculus presented in this paper uses the lock of the object cc to synchronize the input and output requests issued by SequentialProcess objects. When the class CommunCenter receives an input request, it performs the statement

```
Monitor::Enter(cc);
```

to lock the object cc. Assume the class CommunCenter can find an output request that has been saved in the list outputRequests and that matches the input request. If the found output request was issued by a thread that encapsulates the SequentialProcess object sp, the class CommunCenter uses the following code to lock the object sp, change the WaitSleepJoin state of the thread to the Running state so that the thread can resume its computation.

```
Monitor::Enter(sp);
Monitor::Pulse(sp);
Monitor::Exit(sp);
```

Then, the class CommunCenter executes the following statement to release the lock of object cc before it returns.

```
Monitor::Exit(cc);
```

If the class CommunCenter cannot find an output request saved in the list outputRequests to satisfy the input request, it saves the input request into the inputRequests list and executes the following code for the current thread that is executing the input request into the WaitSleepJoin state. Here, we assume the input request was generated from the SequentialProcess object spl.

```
Monitor::Exit(cc);
Monitor::Enter(spl);
Monitor::Wait(spl);
Monitor::Exit(spl);
```

The class CommunCenter uses another static method to handle an output request from SequentialProcess objects. The execution thread of the static method also uses the lock of object cc to synchronize the SequentialProcess objects that issue input and output requests. We use the class Monitor to synchronize the activities of a SequentialProcess object and its child of class CompositeProcess so that only after the child completes its execution, the SequentialProcess parent can continue its execution.

Class Mutex

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An object of class Mutex, defined in namespace System::Threading, can be used to synchronize threads that are created in the same process and in different processes. It can be used to enforce exclusive access to a resource. It does not provide all the wait and pulse methods as defined in the class Monitor. We can invoke methods WaitOne, WaitAny, and WaitAll defined in the class Mutex to wait for a Mutex object or for some Mutex objects to be available before the current thread can continue its execution. A thread releases a Mutex object by invoking the instance method ReleaseMutex of the Mutex object. When a thread finishes normally, it releases the Mutex objects that are held by it so that waiting threads can hold them. In the implementation of the π -calculus presented in this paper, we do not use the class Mutex to synchronize input and output actions. However, if the *composite_process* and *sequential_process* nodes in a hierarchy are implemented with operating system processes, we need to use the class Mutex to synchronize the input and output requests of the nodes.

For example, we can use the statement

```
Mutex* inputMut = new Mutex(true);
```

to create a new Mutex object inputMut. The constructor argument true indicates that the current thread that is executing the above statement holds or owns the newly created Mutex object. Another thread can compete for the ownership of the Mutex object with statement

```
inputMut->WaitOne();
```

A thread that holds the inputMut object can release it by executing statement

```
inputMut->ReleaseMutex();
```

5 THE .NET IMPLEMENTATION OF THE π -CALCULUS

Nodes and Threads

The hierarchy $\Im(P)$ for a given π -calculus process expression P is composed of objects of the three classes CompositeProcess, ChoiceProcess, and SequentialProcess, which implement the *composite_process*, *choice_process*, and *sequential_process* nodes, respectively. To maximize the concurrency of an execution of the π -calculus process, we encapsulate CompositeProcess and SequentialProcess objects in .NET threads, which are instances of the standard class Thread. The starting methods of the threads follow the responsibilities of the *composite_process* and *sequential_process* nodes as described in Section 3. They are detailed as follows.

Class Node is the common base of classes CompositeProcess, ChoiceProcess, and SequentialProcess. It introduces instance variables parent and children for a node to hold its parent and children, respectively, and member methods that manage the parent and children variables. An object cp of the class CompositeProcess keeps the string procExpr, described in Section 3. The thread starting method of the object cp identifies the *choice_process* expressions from the string procExpr using top-level occurrences of the compsoite operator '|' in the string. For each of the *choice_process* expressions, an object dp of class ChoiceProcess is created to process the *choice_process* expression, which is used to initialize the instance variable procExpr of the created object dp. The thread starting method of the object cp adds the created object dp into the children list of cp.

The constructor of an object dp of the class ChoiceProcess divides the *choice_process* expression procExpr held in dp into *sequential_process* expressions, each of which is used to create an object sp of class SequentialProcess. The created object sp is assigned to a thread, which is responsible to process the actions specified in the *sequential_process* expression procExpr.

For example, assume the *composite_process* expression (in x y.out y z.0).0 | out z.0 + out x a. in a w.0 is used to create a CompositeProcess object cp0. The thread starting method of object cp0 will create two children dp1 and dp2 of class ChoiceProcess to handle the *choice process* expressions (in x y.out y z.0). 0 and out z.0 + out x a.in a w.0, respectively. The object dpl will create an object spl of class SequentialProcess to handle the *sequential_process* expression (in x y.out y z.0).0. The object dp2 will create two objects sp2 and sp3 to handle the sequential_process expressions out z. 0 and out x a. in a w. 0, respectively. When the thread starting method of the object sp1 handles the composite action (in x y.out y z.0), it create an object cp1 of class CompositeProcess to handle the *composite_process* expression in x y.out y z.0. The object cpl creates an object dp3 of class ChoiceProcess, which creates an object sp4 of class Sequential Process for processing the sequential process expression in x y.out $y \ge 0$. In summary, a handling of the process expression (in x y.out $y \ge 0$). 0 | out $z \ge 0$ + out x a.in a w.0 needs to create the above mentioned objects, which form the hierarchy shown in Fig. 5.1. Here, we introduce the object names such as cp0 and dp1 for illustration. The source code of the implementation of the π -calculus does not use the object names.

A CompositeProcess object cp such as the objects cp0 and cp1 shown in Fig. 5.1 holds its children, which are objects of class ChoiceProcess, with the instance variable children. If the list children becomes empty and the object cp is the root node of the hierarchy 3, the execution is completed. If the list children in object cp becomes empty but the object cp is not the root, the node cp invokes the removeChild method of its parent object before it completes the execution specified by the thread starting method processing defined in class CompositeProcess.

Similar to a composite process object, a choice process object dp of class ChoiceProcess such as the node dp1, dp2, or dp3 shown in Fig. 5.1 uses a list named children to hold its children, which are objects of class SequentialProcess. If the list children becomes empty, the object dp winds up its computation by invoking the removeChild method of its CompositeProcess parent to remove itself from the hierarchy.



Figure 5.1 A hierarchy for executing a π -calculus process expression

An object sp of class SequentialProcess such as the node sp1, sp2, sp3, or sp4 shown in Fig. 5.1 encapsulates methods for handling the different terms and basic actions specified in the *sequential_process* expression procExpr that is stored in the object sp. The instance methods are described as follows. We shall describe the communication between the SequentialProcess objects in the following subsection.

- The method handle_new() is invoked to handle a new-term that is in the front of the expression procExpr. It adds the names declared by the new-term into the declared-Names list.
- The method handle_repeat() handles a repeat term ! by creating a new SequentialProcess object whose declaredName list and inputValue function has equal values as the same named data structures in the object sp.
- The method handle_match() handles a match-term [n1 = n2] by retrieving the values of the names n1 and n2 in the following way. The method tries to determine if the name n1 is an input parameter in a lowest ancestor sp' of the node sp by searching the function inputValue in the ancestors of sp. If the search succeeds in a SequentialProcess object sp', the desired value is equal to the value inputValue(n1); otherwise, method handle_match() tries to determine if the name n1 is a name declared in a lowest ancestor sp' of the node sp by searching the declaredNames list in the ancestors of sp. If the search succeeds in object sp', the retrieved value is equal to (sp', n1); otherwise, the name n1 is free with respect to the whole hierarchy and the retrieved value is equal to (0, n1). The method handle_match() retrieves a value for the other name n2 similarly. If the two retrieved values have same components, the method handle_match() completes

its evaluation successfully; otherwise, it changes the procExpr expression to the empty string S["] so that the execution of object sp can wind up.

- The method performedAction() is invoked after an input, output, or internal action is performed by the object sp. It simply requests the ChoiceProcess ancestors of the node sp to remove all the children but the SequentialProcess ancestors of the node sp. Thus, the method enforces the choice operations at the ChoiceProcess ancestor nodes of the node sp.
- The method after_action() is invoked after an input, output, or internal action is performed by the object sp. It installs the SequentialProcess nodes stored in the repeatedNodes list in the object sp. Particularly, for each object sp' in the repeated-Nodes list, the method after_action() creates a ChoiceProcess object dp', sets the parent field in dp' with the parent of parent of the node sp, and sets the parent field in object sp' with dp'. It also creates a thread for the object sp'. The method after_action() will empty the repeatedNodes list in the object sp after it adds all the nodes in the list to the hierarchy.
- The method handle_inaction() is invoked if the front action in the process expression procexpr in object sp is the inaction 0. The method simply sets the procexpr variable in node sp with the empty string S"" and returns.
- The method handle_tao() is invoked to handle the front tao action in the procExpr expression in object sp. It simply removes the front action from procExpr and invokes the methods performedAction() and after_action() for object sp.
- The method handle_input() is invoked for object sp if the front action in expression procExpr in sp is an input action. It uses the communication link and parameters shown in the input action expression to invoke the InputRequest() method defined in class CommunCenter, which decides whether the input action can be honored immediately or should be saved in the inputRequests list in the object cc. After the InputRequest() method is executed, the methods performedAction() and after_action() of object sp are executed.
- The method handle_output() of object sp for handling the front output action in expression procExpr of object sp is similar to the above method handle_input(). It uses the communication link and arguments coded in the output action expression to invoke the OutputRequest() method defined in class CommunCenter, which decides whether the output action can be honored immediately or should be saved in the outputRequests list in the object cc.
- The method handle_paren() is invoked if the front action in the expression procExpr in object sp is a parenthesized process expression (R). It creates a new object cp of class CompositeProcess to encapsulate the *composite_process* expression R and creates a thread to execute the processing method for the object cp. It then waits for the object cp to complete its execution by invoking the method Join() of cp.

Process Communication

We use the class CommunCenter to coordinate and synchronize the input and output requests issued by SequentialProcess objects. Particularly, the class uses a static variable cc to hold the unique object of the class CommunCenter for encapsulating the inputRequests and outputRequests lists. The static methods InputRequest() and OutputRequest() defined in the class CommuCenter with signatures

```
void InputRequest(NodeName*, ArrayList*, SequentialProcess*);
void OutputRequest(NodeName*, ArrayList*, SequentialProcess*);
```

are responsible to accept input and output requests from SequentialProcess objects. The first argument of the above methods denotes a communication link in the form (sp, name) such that the pointer 0 for sp indicates name is free in the whole hierarchy and a valid pointer sp indicates name is declared in the node sp with a new-term. The second argument carries either the parameter list or the argument list. The third argument points to the Sequential-Process object that issued the input or output request.

The object cc not only stores the lists inputRequests and outputRequests but also is used to synchronize accesses to the lists by the SequentialProcess objects. Particulaly, the above two static methods must request the lock of object cc before they can access or modify the lists. Thus, we keep the integrity of the lists.

The .NET Executable Code

The .NET implementation of the π -calculus is compiled to an application named pi-calculator.exe for Microsoft Windows. The application allows a user to enter a π -calculus process expression coded in the ASCII language presented in Section 3. For example, after starting the application with command

pi-calculator

we can enter the process expression (in x y. out y z. 0). 0 | out z. 0 + out x a. in a w. 0, which was displayed in Fig. 5.1. The application will display the following information to indicate the communications performed in the execution of the process:

A communication is realized through link: node free and name x: Parameter name: y argument: node free name a; Communication is completed. A communication is realized through link: node free and name a: Parameter name: w argument: node free name z; Communication is completed.

In the first sentence, the term free is used to indicate the channel name x is a free name with respect to the whole composite process expression. The term free is used similarly in the following sentences. The above display shows the second *sequential_process* expression out x a.in a w.0 in the second *choice_process* expression out z.0 + out x a.in a w.0 communicates an output request and an input request with the first *choice_process* expression. Thus, the first *sequential_process* expression out z.0 is rendered void by the activity of the second.

The composite process expression (in x y.out y z.0).0 | out z.0 + new a out x a.in a w.0 introduces a new-term into the above *composite_process* expression. Its execution will display the following information, which indicates that the node 11 declared the name a.

The declared name a is communicated as an argument in the first communication. It is used as the communication link in the second communication.

A communication is realized through link: node free and name x:

Parameter name: y argument: node 11 name a;

Communication is completed.

A communication is realized through link: node 11 and name a:

Parameter name: w argument: node free name z;

Communication is completed.

6 CONCLUSIONS

The π -calculus proposed by Milner et al. [Milner et al. 1989, Milner et al. 1992] has been studied extensively as a theoretical model of mobile communication systems. Here, we reformulate the syntax of the π -calculus and describe how to implement the π -calculus based on the reformulated syntax. The operational semantics of the π -calculus based on the reformulated syntax is realized on the .NET Framework for the construction of a Windows application, which accepts π -calculus process expressions and executes them by dispatching threads.

The presented syntax and semantics of the π -calculus implies that we can realize π -calculus process expressions, which specify the behavious of mobile communication systems, with ditributed systems that are based on the CORBA or Web Services.

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About the author

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With great regret we report the passing of Liwu Li on April 21, 2005 at the age of 58.



Dr. Liwu Li was a professor in School of Computer Science at University of Windsor, Canada. His research interests include objectoriented language design and implementation, object-oriented software analysis and design, and software process design and execution.